



पुर्णमा International School
Shree Swaminarayan Gurukul, Zundal

Grade - 10
Maths
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CHAPTER NO. – 1

CHAPTER NAME – REAL NUMBERS

KEY POINTS TO REMEMBER –

- **Natural Numbers:** Counting Numbers are called Natural Numbers are denoted by
$$N = \{1, 2, 3, 4, 5, \dots\}$$
- **Whole Numbers :** The collection of Natural Numbers along with zero is the collection of Whole Numbers and is denoted by W.
$$W = \{0, 1, 2, 3, 4, \dots\}$$
- **Integers:** The collection of Natural numbers, their negatives along with the number zero are called Integers. This collection is denoted by Z.
$$Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$
- **Rational number:** The numbers, which are obtained by dividing two integers, are called Rational numbers. Division by zero is not defined.
$$Q = \{ p/q: p \text{ and } q \text{ are integers, } q \neq 0\}$$
- **Prime number:** The number other than 1, with only factors namely 1 and the number itself, is a prime number.
$$\{ 2, 3, 5, 7, 11, 13, 17, 19, \dots\}$$
- **Co-prime number:** If HCF of two numbers is 1, then the two numbers are called co-prime.
- **The Fundamental Theorem of Arithmetic**
Every composite number can be expressed (factorized) as a product of primes and this factorization is unique, apart from the order in which the prime factors occur.
$$\text{Ex : } 24 = 2 \times 2 \times 2 \times 3 = 3 \times 2 \times 2 \times 2$$

The Fundamental Theorem of Arithmetic says that every composite number can be factorized as a product of primes.
- **HCF and LCM:**(by prime factorization method)
HCF: Product of the smallest power of each common prime factor in the numbers.
LCM: Product of the greatest power of each common prime factor in the numbers.
- For any two positive integers a and b

$$\text{HCF (a x b) X LCM (a x b) = a x b}$$

- Revisiting Irrational Numbers

Theorem 1.3: Let p be a prime number. If p divides a^2 , then p divides a , Where a is a positive integer.

Theorem 1.4: $\sqrt{2}$ is irrational.

- Revisiting Rational Numbers and Their Decimal Expansions

Theorem 1.5 : Let x be a rational number. Whose decimal expansion terminates then x can be expressed in the form $\frac{p}{q}$. Where p and q are co-prime, and prime factorization of q is of the form $2^m 5^n$, where m, n are non negative integers.

Theorem 1.6: Let $x = \frac{p}{q}$, $q \neq 0$ to be a rational number, such that the prime factorization of q is not of the form $2^m 5^n$, where m, n are non negative integers. Then x has a decimal expansion which terminates.

Theorem 1.7: : Let $x = \frac{p}{q}$, $q \neq 0$ to be a rational number, such that the prime factorization of q is of the form $2^m 5^n$, where m, n are non negative integers. Then x has a decimal expansion which is non-terminating repeating

Example: 1 Express 140 as a product of its prime factor

Solution: $140 = 2 \times 2 \times 5 \times 7$
 $= 2^2 \times 5 \times 7$

Example: 2 Find the HCF and LCM 91 and 26 by prime factorization.

Solution: $26 = 2 \times 13$

$$91 = 7 \times 13$$

$$\text{HCF} = 13$$

$$\text{LCM} = 2 \times 7 \times 13 = 182$$

Example: 3 Find the HCF and LCM 12, 15 and 21 by prime factorization.

Solution: $12 = 2 \times 2 \times 3 = 2^2 \times 3$

$$15 = 3 \times 5$$

$$21 = 3 \times 7$$

$$\text{HCF} = 3$$

$$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$$

Example: 4 Given that $\text{HCF}(306, 657) = 9$, find $\text{LCM}(306, 657)$.

Solution: $\text{HCF}(306, 657) = 9$

We know that, $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$$\text{LCM} = 22338$$

Example: 5 Check whether 6^n can end with the digit 0 for any natural number n .

Solution: If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorization of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorization of 6^n .

Hence, for any value of n , 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n .

Example: 6 Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution: Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$$

$$= 13 \times 78$$

$$= 13 \times 13 \times 6$$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$$

$$= 5 \times (1008 + 1)$$

$$= 5 \times 1009$$

1009 can not be factorized further.

Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Example: 7: Prove that $\sqrt{5}$ is irrational.

Answer : Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

Let a and b have a common factor other than 1. Then we can divide them by the common factor, and assume that a and b are co-prime.

$$a = \sqrt{5}b$$

$$a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that a is divisible by 5.

Let $a = 5k$, where k is an integer

$$(5k)^2 = 5b^2$$

$$b^2 = 5k^2$$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Example: 8 Prove that $3+2\sqrt{5}$ is irrational.

Answer :

Let $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$3+2\sqrt{5} = \frac{a}{b}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{1}{2} \left(\frac{a}{b} - 3 \right)$$

$$\frac{1}{2} \left(\frac{a}{b} - 3 \right)$$

Since a and b are integers, $\frac{a}{b}$ will also be rational

And therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false.

Therefore, $3+2\sqrt{5}$ is irrational.

Example: 9 Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Answer :

(i) $\frac{13}{3125}$

$$3125 = 5^5$$

The denominator is of the form 5^m .

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$$8 = 2^3$$

The denominator is of the form 2^m .

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$$455 = 5 \times 7 \times 13$$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$$1600 = 2^6 \times 5^2$$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

WORK SHEET

1. Using prime factorization , find the HCF of
 - (i) 405 and 2520
 - (ii) 504 and 1188
 - (iii) 960 and 1575
2. Using prime factorization, find the HCF and LCM of:
 - (i) 36 and 84
 - (ii) 23 and 31
 - (iii) 96 and 404
 - (iv) 144 and 198
 - (v) 396 and 1080

In each case, verify that $\text{HCF} \times \text{LCM} = \text{product of given number}$

3. Using prime factorization, find the HCF and LCM of:
 - (i) 8, 9 and 25
 - (ii) 12 ,15 and 21
 - (iii) 17,23 and 29
 - (iv) 24, 36 and 40
 - (v) 30, 72 and 432
4. The HCF of two number is 23 and their LCM is 1449. If one of the number is 161, find the other.
5. The HCF of two number is 145 and their LCM is 2175. If one of the number is 725, find the other.
6. The HCF of two number is 18 and their product is 12960. Find their LCM.
7. State Euclid's division lemma.
8. State whether the given statement is true or false.
 - (i) The sum of two rational is always rational.
 - (ii) The product of two rational is always rational.
 - (iii) The sum of two irrational is always an irrational.
 - (iv) The product of two irrational is always an irrational.
 - (v) The sum of rational and an irrational is always irrational.
 - (vi) The product of rational and an irrational is always irrational.

CHAPTER 7

COORDINATES GEOMETRY

KEY POINTS TO REMEMBER –

- DISTANCE FORMULA
- SECTION FORMULA

1. **Distance Formula:** The length of a line segment joining A and B is the distance between two points A (x_1, y_1) and B (x_2, y_2)

THE DISTANCE FORMULA

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. The distance of a point P (x, y) from the origin $(0, 0)$ is

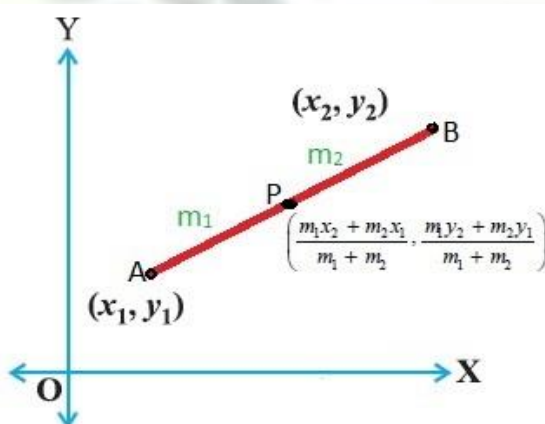
O $(0, 0)$

P (x, y)

$$OP = \sqrt{(x - 0)^2 + (y - 0)^2}$$

$$OP = \sqrt{x^2 + y^2}$$

3. **SECTION FORMULA:** The coordinate of the point P (x, y) which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m_1: m_2$ are



Section Formula

So, the coordinates of the point $P(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$, internally, in the ratio $m_1: m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_2 + m_1}, \frac{m_1y_2 + m_2y_1}{m_2 + m_1} \right)$$

This is known as the **section formula**.

4. MID-POINT FORMULA: The midpoint of the line segment joining the points A (x_1, y_1) and B (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Chapter - 7
Coordinate Geometry
Exercise 7.1

= Find the distance between the following pairs of points:

(i) (2, 3), (4,1)

(ii) (-5, 7), (-1, 3)

(iii) (a, b), (-a, b)

Ans. (i) Applying Distance Formula to find distance between points (2, 3) and (4,1), we get d =

$$\sqrt{(4-2)^2 + (1-3)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$

= Applying Distance Formula to find distance between points (-5, 7) and (-1, 3), we get d =

$$\sqrt{[-1-(-5)]^2 + (3-7)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ units}$$

= Applying Distance Formula to find distance between points (a, b) and (-a, -b), we get

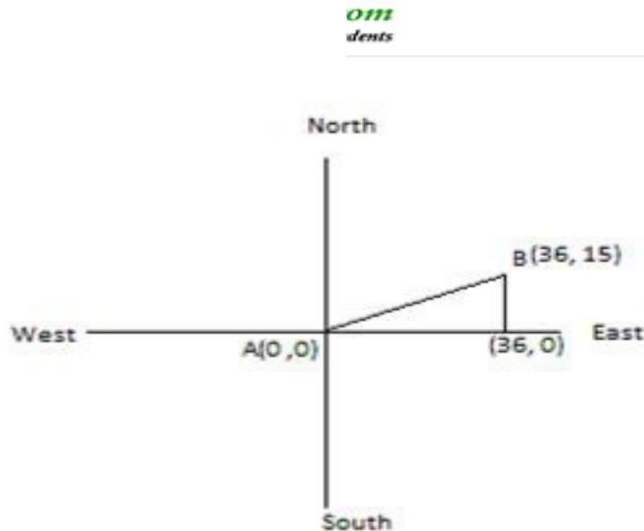
$$d = \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} = \sqrt{4(a^2 + b^2)} = 2\sqrt{a^2 + b^2}$$

2. Find the distance between the points (0, 0) and (36, 15). Also, find the distance between towns A and B if town B is located at 36 km east and 15 km north of town A.

Ans. Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get

$$d = \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39 \text{ units}$$

Town B is located at 36 km east and 15 km north of town A. So, the location of town A and B can be shown as:



Clearly, the coordinates of point A are (0, 0) and coordinates of point B are (36, 15).

To find the distance between them, we use Distance formula:

$$d = \sqrt{[36-0]^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39 \text{ Km}$$

3. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Ans. Let A = (1, 5), B = (2, 3) and C = (-2, -11)

Using Distance Formula to find distance AB, BC and CA.

$$AB = \sqrt{[2-1]^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$BC = \sqrt{[-2-2]^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$$

$$CA = \sqrt{[-2-1]^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$$

Since $AB + AC \neq BC$, $BC + AC \neq AB$ and $AC \neq BC$.

Therefore, the points A, B and C are not collinear.

4. Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Ans. Let $A = (5, -2)$, $B = (6, 4)$ and $C = (7, -2)$

Using Distance Formula to find distances AB, BC and CA.

$$AB = \sqrt{[6 - 5]^2 + [4 - (-2)]^2} = \sqrt{(1)^2 + (6)^2} = \sqrt{1 + 36} = \sqrt{37}$$

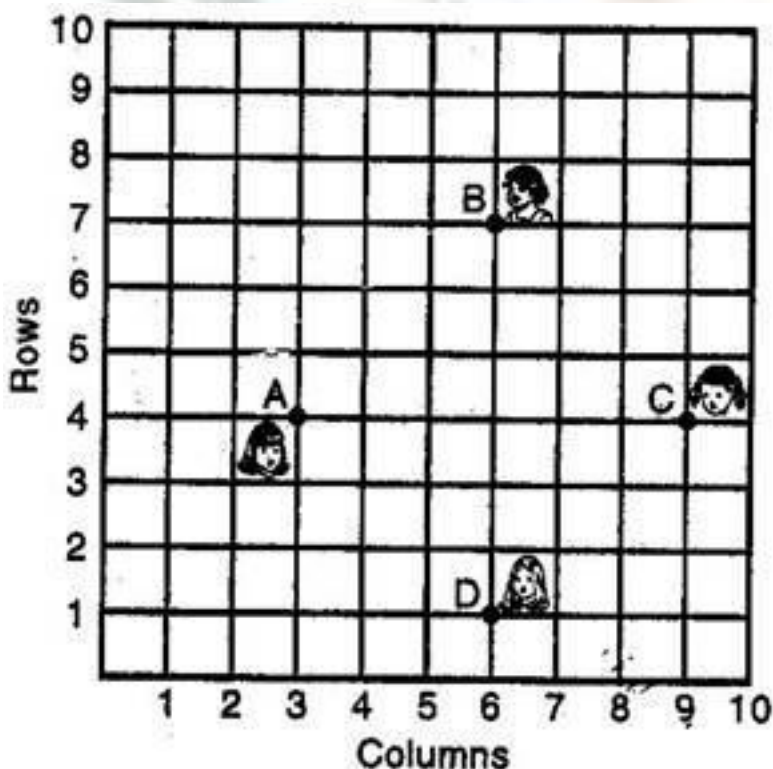
$$BC = \sqrt{[7 - 6]^2 + (-2 - 4)^2} = \sqrt{(1)^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37}$$

$$CA = \sqrt{[7 - 5]^2 + [-2 - (-2)]^2} = \sqrt{(2)^2 + (0)^2} = \sqrt{4 + 0} = \sqrt{4} = 2$$

Since $AB = BC$.

Therefore, A, B and C are vertices of an isosceles triangle.

(v) — In a classroom, 4 friends are seated at the points A (3, 4), B (6, 7), C (9, 4) and D (6, 1). Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli. "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



Ans. We have A = (3, 4), B = (6, 7), C = (9, 4) and D = (6, 1)

Using Distance Formula to find distances AB, BC, CD and DA, we get AB =

$$\sqrt{[6-3]^2 + [7-4]^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{[9-6]^2 + [4-7]^2} = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{[6-9]^2 + [1-4]^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$DA = \sqrt{[6-3]^2 + [1-4]^2} = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

Therefore, All the sides of ABCD are equal here. ... (1)

Now, we will check the length of its diagonals.

$$AC = \sqrt{[9-3]^2 + [4-4]^2} = \sqrt{(6)^2 + (0)^2} = \sqrt{36+0} = 6$$

$$BD = \sqrt{[6-6]^2 + [1-7]^2} = \sqrt{(0)^2 + (-6)^2} = \sqrt{0+36} = \sqrt{36} = 6$$

So, Diagonals of ABCD are also equal. ... (2)

From (1) and (2), we can definitely say that ABCD is a square.

Therefore, Champa is correct.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(i) (-1, -2), (1, 0), (-1, 2), (-3, 0)

(ii) (-3, 5), (3, 1), (0, 3), (-1, -4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Ans. (i) Let A = (-1, -2), B = (1, 0), C = (-1, 2) and D = (-3, 0)

Using Distance Formula to find distances AB, BC, CD and DA, we get

$$AB = \sqrt{[1 - (-1)]^2 + [0 - (-2)]^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{[-1 - 1]^2 + [2 - 0]^2} = \sqrt{(-2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{[-3 - (-1)]^2 + [0 - 2]^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$DA = \sqrt{[-3 - (-1)]^2 + [0 - (-2)]^2} = \sqrt{(-2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

Therefore, all four sides of quadrilateral are equal. ... (1)

Now, we will check the length of diagonals.

$$AC = \sqrt{[-1 - (-1)]^2 + [2 - (-2)]^2} = \sqrt{(0)^2 + (4)^2} = \sqrt{0 + 16} = \sqrt{16} = 4$$

$$BD = \sqrt{[-3 - 1]^2 + [0 - 0]^2} = \sqrt{(-4)^2 + (0)^2} = \sqrt{16 + 0} = \sqrt{16} = 4$$

Therefore, diagonals of quadrilateral ABCD are also equal. ... (2)

From (1) and (2), we can say that ABCD is a square.

(ii) Let A = (-3, 5), B = (3, 1), C = (0, 3) and D = (-1, -4)

Using Distance Formula to find distances AB, BC, CD and DA, we get

$$AB = \sqrt{[3 - (-3)]^2 + [1 - 5]^2} = \sqrt{(6)^2 + (-4)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{[0 - 3]^2 + [3 - 1]^2} = \sqrt{(-3)^2 + (2)^2} = \sqrt{9 + 4} = \sqrt{13}$$

$$CD = \sqrt{[-1 - 0]^2 + [-4 - 3]^2} = \sqrt{(-1)^2 + (-7)^2} = \sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$$

$$DA = \sqrt{[-1 - (-3)]^2 + [-4 - 5]^2} = \sqrt{(2)^2 + (-9)^2} = \sqrt{4 + 81} = \sqrt{85}$$

We cannot find any relation between the lengths of different sides.

Therefore, we cannot give any name to the quadrilateral ABCD

(iii) Let A = (4, 5), B = (7, 6), C = (4, 3) and D = (1, 2)

Using Distance Formula to find distances AB, BC, CD and DA, we get AB =

$$\sqrt{[7 - 4]^2 + [6 - 5]^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9 + 1} = \sqrt{10}$$

$$BC = \sqrt{[4 - 7]^2 + [3 - 6]^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{[1 - 4]^2 + [2 - 3]^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10}$$

$$DA = \sqrt{[1 - 4]^2 + [2 - 5]^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$$

Here opposite sides of quadrilateral ABCD are equal. ... (1)

We can now find out the lengths of diagonals.

$$AC = \sqrt{[4 - 4]^2 + [3 - 5]^2} = \sqrt{(0)^2 + (-2)^2} = \sqrt{0 + 4} = \sqrt{4} = 2$$

$$BD = \sqrt{[1 - 7]^2 + [2 - 6]^2} = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$$

Here diagonals of ABCD are not equal. ... (2)

From (1) and (2), we can say that ABCD is not a rectangle therefore it is a parallelogram.

7. Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

Ans. Let the point be (x, 0) on x-axis which is equidistant from (2, -5) and (-2, 9). Using

Distance Formula and according to given conditions we have:

$$\sqrt{[x-2]^2 + [0-(-5)]^2} = \sqrt{[x-(-2)]^2 + [(0-9)]^2}$$

$$\Rightarrow \sqrt{x^2 + 4 - 4x + 25} = \sqrt{x^2 + 4 + 4x + 81}$$

Squaring both sides, we get

$$\Rightarrow x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$\Rightarrow -4x + 29 = 4x + 85$$

$$\Rightarrow 8x = -56$$

$$\Rightarrow x = -7$$

Therefore, point on the x-axis which is equidistant from (2, -5) and (-2, 9) is (-7, 0)

8. Find the values of y for which the distance between the points P (2, -3) and Q (10, y) is 10units.

Ans. Using Distance formula, we have

$$10 = \sqrt{(2-10)^2 + (-3-y)^2}$$

$$\Rightarrow 10 = \sqrt{(-8)^2 + 9 + y^2 + 6y}$$

$$\Rightarrow 10 = \sqrt{64 + 9 + y^2 + 6y}$$

Squaring both sides, we get

$$100 =$$

$$\Rightarrow y^2 + 6y - 27 = 0$$

Solving this Quadratic equation by factorization, we can write

$$\Rightarrow y^2 + 9y - 3y - 27 = 0$$

$$\Rightarrow y(y + 9) - 3(y + 9) = 0$$

$$\Rightarrow (y + 9)(y - 3) = 0$$

$$\Rightarrow y = 3, -9$$

9. If, Q (0, 1) is equidistant from P (5, -3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

Ans. It is given that Q is equidistant from P and R. Using Distance Formula, we get $PQ=RQ$

$$\Rightarrow \sqrt{(0 - 5)^2 + [1 - (-3)]^2} = \sqrt{(0 - x)^2 + (1 - 6)^2}$$

$$\Rightarrow \sqrt{(-5)^2 + (4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

$$\Rightarrow \sqrt{25 + 16} = \sqrt{x^2 + 25}$$

Squaring both sides, we get

$$\Rightarrow 25 + 16 = x^2 + 25$$

$$\Rightarrow x^2 = 16$$

$$\Rightarrow x = 4, -4$$

Thus, Q is (4, 6) or (-4, 6).

Using Distance Formula to find QR, we get

$$\text{Using value of } x = 4 \text{ QR} = \sqrt{(4 - 0)^2 + (6 - 1)^2} = \sqrt{16 + 25} = \sqrt{41}$$

$$\text{Using value of } x = -4 \text{ QR} = \sqrt{(-4 - 0)^2 + (6 - 1)^2} = \sqrt{16 + 25} = \sqrt{41}$$

$$\text{Therefore, QR} = \sqrt{41}$$

Using Distance Formula to find PR, we get

$$\text{Using value of } x = 4 \text{ PR} = \sqrt{(4 - 5)^2 + [6 - (-3)]^2} = \sqrt{1 + 81} = \sqrt{82}$$

$$\text{Using value of } x = -4 \text{ PR} = \sqrt{(-4 - 5)^2 + [6 - (-3)]^2} = \sqrt{81 + 81} = \sqrt{162} = 9\sqrt{2}$$

Therefore, $x = 4, -4$

$$\text{QR} = \sqrt{41}, \text{PR} = \sqrt{82}, 9\sqrt{2}$$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.

Ans. It is given that (x, y) is equidistant from $(3, 6)$ and $(-3, 4)$.

Using Distance formula, we can write

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{[x-(-3)]^2 + (y-4)^2}$$

$$\Rightarrow \sqrt{x^2 + 9 - 6x + y^2 + 36 - 12y} = \sqrt{x^2 + 9 + 6x + y^2 + 16 - 8y}$$

Squaring both sides, we get

$$\Rightarrow x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$\Rightarrow -6x - 12y + 45 = 6x - 8y + 25$$

$$\Rightarrow 12x + 4y = 20$$

$$3x + y = 5$$

Exercise 7.2

1. Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio $2:3$.

Ans. Let $x_1 = -1, x_2 = 4, y_1 = 7$ and $y_2 = -3, m_1 = 2$ and $m_2 = 3$

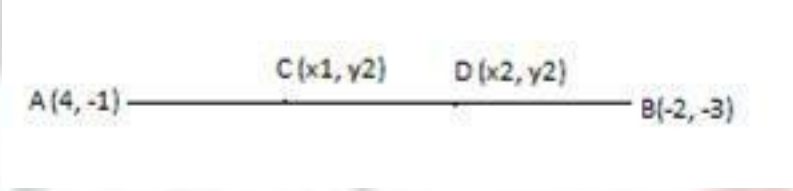
Using Section Formula to find coordinates of point which divides join of $(-1, 7)$ and $(4, -3)$ in the ratio 2:3, we get

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2 \times 4 + 3 \times (-1)}{2 + 3} = \frac{8 - 3}{5} = \frac{5}{5} = 1$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2 \times (-3) + 3 \times 7}{2 + 3} = \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Therefore, the coordinates of point are $(1, 3)$ which divides join of $(-1, 7)$ and $(4, -3)$ in the ratio 2:3.

2. Find the coordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.

Ans. 

We want to find coordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.

We are given $AC = CD = DB$

We want to find coordinates of point C and D.

Let coordinates of point C be (x_1, y_1) and let coordinates of point D be (x_2, y_2) .

Clearly, point C divides line segment AB in 1:2 and point D divides line segment AB in 2:1.

Using Section Formula to find coordinates of point C which divides join of $(4, -1)$ and $(-2, -3)$

in the ratio 1:2, we get

$$x_1 = \frac{1 \times (-2) + 2 \times 4}{1 + 2} = \frac{-2 + 8}{3} = \frac{6}{3} = 2$$

$$y_1 = \frac{1 \times (-3) + 2 \times (-1)}{1 + 2} = \frac{-3 - 2}{3} = \frac{-5}{3}$$

Using Section Formula to find coordinates of point D which divides join of $(4, -1)$ and $(-2, -3)$ in the ratio

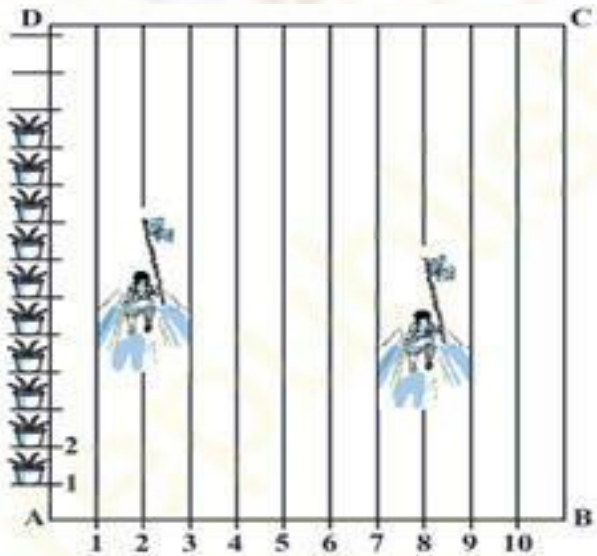
2:1, we get

$$x_2 = \frac{2 \times (-2) + 1 \times 4}{1 + 2} = \frac{-4 + 4}{3} = \frac{0}{3} = 0$$

$$y_2 = \frac{2 \times (-3) + 1 \times (-1)}{1 + 2} = \frac{-6 - 1}{3} = \frac{-7}{3}$$

Therefore, coordinates of point C are $(2, -5/3)$ and coordinates of point D are $(0, -7/3)$

3. To conduct sports day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD. Niharika runs 14th of the distance AD on the 2nd line and posts a green flag. Preet runs 15th of the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segments joining the two flags, where should she post her flag?



Ans. Niharika runs 14th of the distance AD on the 2nd line and posts a green flag.

There are 100 flower pots. It means, she stops at 25th flower pot.

Therefore, the coordinates of point where she stops are (2 m, 25 m).

Preet runs 15th of the distance AD on the eighth line and posts a red flag. There are 100 flower pots. It means, she stops at 20th flower pot.

Therefore, the coordinates of point where she stops are (8, 20).

Using Distance Formula to find distance between points (2 m, 25 m) and (8 m, 20 m), we get

$$d = \sqrt{(2-8)^2 + (25-20)^2} = \sqrt{(-6)^2 + 5^2} = \sqrt{36+25} = \sqrt{61}m$$

Rashmi posts a blue flag exactly halfway the line segment joining the two flags. Using

section formula to find the coordinates of this point, we get

$$x = \frac{2+8}{2} = \frac{10}{2} = 5$$

$$y = \frac{25+20}{2} = \frac{45}{2}$$

Therefore, coordinates of point, where Rashmi posts her flag are $(5, \frac{45}{2})$.

It means she posts her flag in 5th line after covering $\frac{45}{2} = 22.5$ m of distance.

4. Find the ratio in which the line segment joining the points $(-3, 10)$ and $(6, -8)$ is divided by $(-1, 6)$.

Ans. Let $(-1, 6)$ divides line segment joining the points $(-3, 10)$ and $(6, -8)$ in $k:1$.

Using Section formula, we get

$$-1 = \frac{(-3) \times 1 + 6 \times k}{k+1}$$

$$-k - 1 = (-3 + 6k)$$

$$-7k = -2$$

$$k = \frac{2}{7}$$

Therefore, the ratio is $\frac{2}{7}:1$ which is equivalent to $2:7$.

Therefore, $(-1, 6)$ divides line segment joining the points $(-3, 10)$ and $(6, -8)$ in $2:7$.

5. Find the ratio in which the line segment joining A $(1, -5)$ and B $(-4, 5)$ is divided by the x-axis. Also find the coordinates of the point of division.

Ans. Let the coordinates of point of division be $(x, 0)$ and suppose it divides line segment joining A $(1, -5)$ and B $(-4, 5)$ in $k:1$.

According to Section formula, we get

$$x = \frac{1 \times 1 + (-4) \times k}{k+1} = \frac{1-4k}{k+1} \text{ and } 0 = \frac{(-5) \times 1 + 5k}{k+1} \dots (1)$$

$$0 = \frac{(-5) \times 1 + 5k}{k+1}$$

$$5 = 5k$$

$$k = 1$$

Putting value of k in (1), we get

$$x = \frac{1 \times 1 + (-4) \times 1}{1+1} = \frac{1-4}{2} = \frac{-3}{2}$$

Therefore, point $\left(\frac{-3}{2}, 0 \right)$ on x-axis divides line segment joining A (1, -5) and B (-4, 5) in 1:1.

6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Ans. Let A = (1, 2), B = (4, y), C = (x, 6) and D = (3, 5)

We know that diagonals of parallelogram bisect each other. It means that coordinates of midpoint of diagonal AC would be same as coordinates of midpoint of diagonal BD. ... (1)

Using Section formula, the coordinates of midpoint of AC are:

$$\frac{1+x}{2}, \frac{2+6}{2} = \frac{1+x}{2}, 4$$

Using Section formula, the coordinates of midpoint of BD are:

$$\frac{4+3}{2}, \frac{5+y}{2} = \frac{7}{2}, \frac{5+y}{2}$$

According to condition (1), we have

$$\frac{1+x}{2} = \frac{7}{2}$$

$$(1+x) = 7$$

$$x = 6$$

Again, according to condition (1), we also have

$$4 = \frac{5+y}{2}$$

$$8 = 5+y$$

$$y = 3$$

Therefore, $x = 6$ and $y = 3$

7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).

Ans. We want to find coordinates of point A. AB is the diameter and coordinates of center are (2, -3) and, coordinates of point B are (1, 4).

Let coordinates of point A are (x, y). Using section formula, we get

$$2 = \frac{x+1}{2}$$

$$4 = x + 1$$

$$x = 3$$

Using section formula, we get

$$-3 = \frac{4+y}{2}$$

$$-6 = 4 + y$$

$$y = -10$$

Therefore, Coordinates of point A are (3, -10).

8. If A and B are (-2, -2) and (2, -4) respectively, find the coordinates of P such that $AP = \frac{3}{7} AB$ and P lies on the line segment AB.

Ans. A = (-2, -2) and B = (2, -4)



It is given that $AP = \frac{3}{7} AB$

$$PB = AB - AP = AB - \frac{3}{7} AB = \frac{4}{7} AB$$

So, we have AP: PB = 3: 4

Let coordinates of P be (x, y)

Using Section formula to find coordinates of P, we get

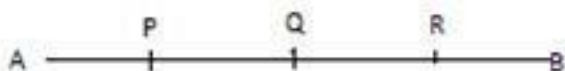
$$x = \frac{(-2) \times 4 + 2 \times 3}{3 + 4} = \frac{6 - 8}{7} = \frac{-2}{7}$$

$$y = \frac{(-2) \times 4 + (-4) \times 3}{3 + 4} = \frac{-8 - 12}{7} = \frac{-20}{7}$$

Therefore, Coordinates of point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

9. Find the coordinates of the points which divides the line segment joining A (-2, 2) and B (2, 8) into four equal parts.

Ans. A = (-2, 2) and B = (2, 8)



Let P, Q and R are the points which divide line segment AB into 4 equal parts.

Let coordinates of point $P = (x_1, y_1)$, $Q = (x_2, y_2)$ and $R = (x_3, y_3)$

We know $AP = PQ = QR = RS$.

It means, point P divides line segment AB in 1:3.

Using Section formula to find coordinates of point P, we get

$$x_1 = \frac{(-2) \times 3 + 2 \times 1}{1+3} = \frac{-6+2}{4} = \frac{-4}{4} = -1$$
$$y_1 = \frac{2 \times 3 + 8 \times 1}{1+3} = \frac{6+8}{4} = \frac{14}{4} = \frac{7}{2}$$

Since, $AP = PQ = QR = RS$. It means, point Q is the mid-point of AB.

Using Section formula to find coordinates of point Q, we get

$$x_2 = \frac{(-2) \times 1 + 2 \times 1}{1+1} = \frac{-2+2}{2} = \frac{0}{2} = 0$$
$$y_2 = \frac{2 \times 1 + 8 \times 1}{1+1} = \frac{2+8}{2} = \frac{10}{2} = 5$$

Because, $AP = PQ = QR = RS$.

It means, point R divides line segment AB in 3:1

Using Section formula to find coordinates of point P, we get

$$x_3 = \frac{(-2) \times 1 + 2 \times 3}{1+3} = \frac{-2+6}{4} = \frac{4}{4} = 1$$
$$y_3 = \frac{2 \times 1 + 8 \times 3}{1+3} = \frac{2+24}{4} = \frac{26}{4} = \frac{13}{2}$$

Therefore, $P = (-1, \frac{7}{2})$, $Q = (0, 5)$ and $R = (1, \frac{13}{2})$

4. The distance of the point (3, 4) from x- axis is
(a) 3 (b) 1 (c) 7 (d) 4

5. Find the distance between the point (2, 3) and (4, 5)

(a) 3 (b) $\sqrt{8}$ (c) 5 (d) 4

6. If A (x, 2), b (-3, -4) and C (7, -5) are collinear, than find the value of x

7. Find the point on x-axis which is equidistance from points (-1, 0) and (5, 0)

8. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

SOLVE (EACH CARRY TWO MARKS)

9. Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

10. Find the point on the x-axis which is equidistant from (2, -5) and (-2,9).

11. Find the values of y for which the distance between the points P (2, -3) and Q (10,y)
Is 10 units

12. If, Q (0, 1) is equidistant from P (5, -3) and R (x, 6), find the values of x. Also, find
the distances QR and PR.

13. Find a relation between x and y such that the point (x, y) is equidistant from the
Point(3, 6) and (-3, 4).

SOLVE (EACH CARRY THREE MARKS)

14. Find the coordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2:3.

15. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3)

16. Find the ratio in which the line segment joining the points (-3, 10) and (6, -8) is divided by (-1, 6).
